Motion Planning and Control Under Uncertainty While Sensing the Environment

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Abstract

This paper presents the details of a motion control system we implemented and tested on a mobile robot equipped with various sensor devices. The control system exploits the robot resources to extract geometric features from the environment. These are then used to estimate uncertainty envelopes constraining the location and orientation of the robot. As this knowledge is derived from noisy (possibly competitive) data, we developed an algorithm that fuses uncertainty envelopes derived from multiple sensors to maintain robust estimates of the robot location and orientation in time. The method we propose is inspired by the set membership principle. It refers to a class of techniques for estimating parameters of linear systems under information that constrains the solution to certain sets. Errors are assumed to be bounded so that a simple, effective geometric tool is provided for the data fusion problem.

1 Introduction

The design of a mobile robot able to move autonomously and safely inside a known workspace is a complex task. Undetected motions inherent to the physics of the actuators make the robot drift in time. A way to enhance the reliability of the motion perception may be provided by combining the basic odometric sensors of the robot with more accurate sensing devices that perceive the same motion. Hence, the problem of controlling the robot movements comes to that of fusing noisy data from various sensors.

The method we propose to handle uncertain information is original in the sense that it is not based on the classic approach. The latter usually applies probabilistic methods and a data fusion algorithm essentially derived from the Kalman filter formalism. Instead, we investigated a different approach, inspired by the set membership principle [1] [8] [9] that assumes bounded errors and provides simple geometric tools for the fusion problem. Under this approach, each measurement of a feature is described by an uncertainty envelope (or set) in the configuration space (C-space) that can be represented by a convex polytope or an ellipsoid [9]. We define a *constraint* as the uncertainty set introduced by a new measurement of the robot configuration - the feature - that may be degenerated if the information is only partial. The fusion process is then essentially an intersection operation in the C-space of all the uncertainty sets describing the same feature. Based on this approach, we developed a configuration estimation algorithm (C-estimator).

Let us define an ***observation*** as the set of obstacles observable from a particular configuration in a sequence composing a path (C-path). The ability for the C-estimator to reduce the drift of the robot when moving along the C-path depends on the accuracy of the devices used to sense the environment and varies for different observations. A recent class of planners have been developed that actually take into account the uncertainty of the robot resources to build a C-path. Throughout this paper we will refer to a motion planner that was suggested and investigated by Takeda and Latombe [7]: the *Sensory Uncertainty Field* (SUF) motion path planner. It uses a description of the robot sensor devices and the

map of the workspace to compute the uncertainty value of fields over a discretized C-space. A C-path is then defined across the fields that minimizes the sum of the uncertainty values along it.
We believe that a good collaboration between the motion path planner and the path tracker (both dealing with uncertainties in the system) leads to a reliable motion control of the robot. It is the main concern of this paper to describe the importance of this relationship and to provide experimental results illustrating it. The following section reviews previous works on various approaches for the description of uncertain information and the problem of fusing data from multiple sensors. Section 3 overviews the motion control system. Section 4 details the path tracker. Section 5 presents and discusses the results of four tests comparing various approaches for the motion control system. Finally, Section 6 concludes this paper.

2 Related work

Crowley [3] uses a technique based on the explicit description of the position and sensing uncertainties that maintains an update of the free space of the robot. A Kalman filter form update formula is used to maintain a consensus on the robot configuration in time. Zhang and Faugeras [10] present a model for the representation and combination of 3D line segments and their uncertainties, measured at different times. They solve the fusion problem by applying a modified minimum-variance estimator form of the Kalman Filter. More generally, a brief survey of criteria for fusing competitive information under a probabilistic approach is presented by Sabater and Thomas [1].
The latter reference, continuing previous work from Nakamura and Xu [2], Porill [4] and others present a different approach for the fusion problem : each time a new set of data is sensed, a constraint on the current consensus of a feature is defined in the parameter space. A constraint may have an infinite uncertainty in one or more degrees of freedom, in which case it does not strictly describe the consensus. The particular case of a constraint with only one degree with infinite uncertainty is a *strip* (degenerated case of an ellipsoid). Similarly, the uncertainty envelope of the consensus in the parameter space is an ellipsoid. Since there is no assumption about the probability distribution inside an uncertainty set, any point inside it is a valid estimation for the consensus. The advantages of this approach, among others, is a natural elimination of inconsistencies in the data and early detection and handling of data that do not reduce the uncertainty of the system.
Among works on motion path planner dealing with uncertainty, the most interesting approach is the one that takes into account the uncertainty throughout the entire planning process. Takeda and Latombe in [7] propose to plan the motions of a mobile robot in a Sensory Uncertainty Field (SUF). Given a model of the robot resources and environment, the SUF is computed over the robot parameter space, to predict the uncertainty in all possible configurations that would be computed by matching the data given by the robot sensors against the model of the environment. A planner then uses the SUF to generate a path that minimizes expected errors, so that its execution can be reliably monitored by the sensors.

3 Motion Control System overview

The motivation for a motion control system dealing with uncertainty is to provide a way to move a robot reliably inside a known environment, by planning the motion and controlling the actuators accordingly. Our path tracker controller is based on a simple navigation technique used by aviators, known as ***waypoints navigation*** (or ***dead-reckoning***). The idea is to move the robot along a straight line between the configuration points (waypoints) composing the C-path. When the robot reaches a waypoint, data on the local environment is perceived and interpreted to estimate a consensus on the robot configuration. Let us introduce some notation for the possible representation of the robot configuration at time t : we denote X^t the *actual* robot configuration, $\hat{X}^t$ the *consensus* on X^t as estimated by the C-estimator,

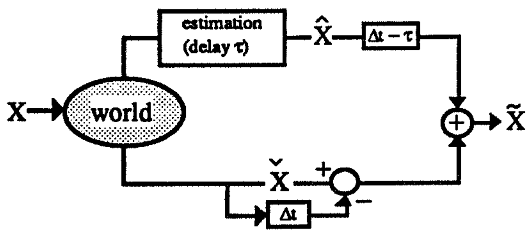

Figure 1: The result of the robot configuration estimation, based on the information extracted from the various sensors, is available at Δt intervals. A maintenance using odometric sensors (during the interval) is thus necessary to update the robot configuration between two estimation processes.

$\tilde{X}^t$ the same consensus updated in time and $\check{X}^t$ the configuration as measured by the robot odometers. The consensus on X^t is modeled by

$$\hat{X}^t = X^t + W^t \tag{1}$$

Under the probabilistic approach, the additive perturbation W^t is generally assumed to be an unbiased random Gaussian variable. Instead, we assume only bounded errors in the measurements. Hence, W^t leads to an uncertainty set in the C-space, centered on $\hat{X}^t$, within which X^t is assumed to be.
Reasonably, the estimation process is not instantaneous. Let us suppose an estimation is started at time t. The new consensus $\hat{X}$ is only available after a delay τ, during which the robot possibly moved. A maintenance of the consensus is thus necessary to properly update the robot configuration in time. The odometric sensors can be used to measure the movement occuring while no new consensus is available. This is formalized by :

$$\tilde{X}^{t+\tau} = \hat{X}^t + \Delta\check{X}^{t,t+\tau}$$

$$\Delta\check{X}^{t,t+\tau} = \check{X}^{t+\tau} - \check{X}^t$$

The use of odometric sensors to determine $\Delta\check{X}^{t,t+\tau}$ introduces an additive error $\Delta\varepsilon$ in the updated consensus. However, τ being relatively small, we can assume that if a new estimation is started at time $t+\tau$, we have $\Delta\varepsilon \leq W^{t+\tau}$. In other terms, performing a new estimation at time $t+\tau$ does not reduce the uncertainty on the robot configuration, unless a sensor device more accurate (on a short distance) than the odometric one is used, which is not our case. Therefore, the distance separating two new estimations is chosen so that $\Delta\varepsilon > W^{t+\Delta t}$, where Δt is the time spent moving between two estimations (see Figure 1). This is satisfied for all the waypoints composing a SUF C-path.

The consensus $\hat{X}^t$ is evaluated from the interpretation of the raw sensed data available at time t and the current knowledge of the robot configuration and uncertainty. However, most of the sensors available as off-the-shelf devices for mobile robots do not provide a direct measurement of the configuration. Instead, features of the local environment are returned, from which an information on the configuration must be derived (assuming a model of the environment is known). Let us denote $s^{s,t}$ the vector composed by the raw data of sensor s, at time t. The core of the C-estimator algorithm performs the following steps:

1. Derive measurements $x_i^{s,t}$ of X^t from $s^{s,t}$, using a description of the workspace.
2. Define a set of constraints C_i^t (in the C-space) on X^t from $x_i^{s,t}$, using a model of the sensor errors.
3. Fuse the C_i^t together with the current uncertainty set of $\tilde{X}^t$, to provide a new consensus $\hat{X}^t$ and a new uncertainty set (smaller or equal in size to the smallest constraining set).

These steps are developed in the next chapter.

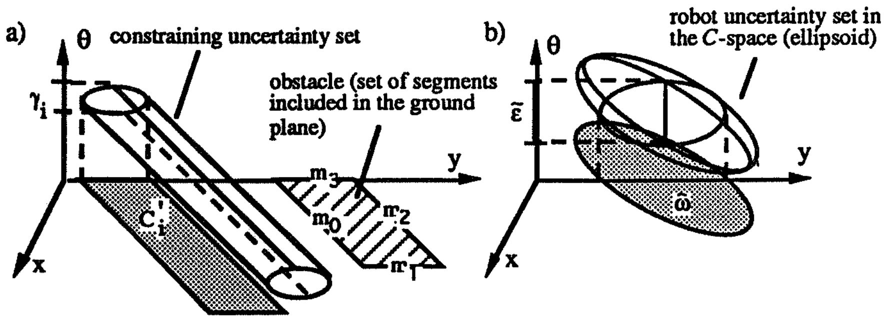

Figure 2: Approximation of the uncertainty regions in the configuration space by their 2D projection on the Cartesian plane and 1D projection on the orientation axis.

4 Deriving constraining sets from range data and fusing them

We describe here the process of interpreting data to create constraints on the consensus of the robot configuration for the particular case of a range sensor. The fusion process that combine these constraining sets with the current knowledge on the robot configuration is detailed. Other types of sensors may need dedicated interpretation processes depending on the type of information they provide. The coverage of more sensor devices, however, is outside the scope of this paper. .

4.1 From range data to constraining uncertainty sets

In a first step, a segmentation of the set of range data (available at the time the estimation is started) to a set of least-square lines is performed using the map of the workplace, the sensor error model and the consensus on the current robot configuration. We call $\mathcal{M}$ the map of the workpsace, m_i a segment composing it and $\mathcal{F}$ the mapping function that project each element of the sensed vector s^s (composed of the range data) to a segment m_i. The principle of the algorithm is the following:

1. Distribute a set $\mathcal{X}$ of configurations over the uncertainty set of $\tilde{X}$. N is the resolution of this discretization.
2. Predict a sensed vector s_j for each configuration X_j of $\mathcal{X}$ $(j = 1, \ldots, N)$, using a model of the sensor errors and the map $\mathcal{M}$.
3. Select the s_j that minimizes $|s_j - s^s|$. The corresponding X_j is the predicted robot configuration.

Trying different configurations over the uncertainty ellipsoid to solve the matching problem is essential, since any candidate inside it may be the actual robot configuration. By properly distributing the X_j's over the uncertainty set, typical mismatch errors due to data points lying nearby a corner or an edge can be considerably reduced.

A form of least square (*LS*) fitting [7] is then applied on s^s to obtain a set of segmented *LS*-lines. For each matching between a *LS*-line l_i and a segment m_i of the map, there is a function $\mathcal{H}$ so that

$$\mathcal{H} : \Re^2 \rightarrow \Re^3 \,;\, \mathcal{H}(l^s, m)_i = C_i \,,$$

where C_i is a constraining uncertainty set in the *C*-space (see Figure 2).

The assumption about the constant uncertainty along l_i fixes C_i parallel to the ground. We consequently approximate C_i by its projections on the ground plane and the orientation axis of the *C*-space. This has some consequences that will be issued further. Under this approximation, we describe a constraint *strip* C_i' in the ground plane (projection of C_i) by the set of points in $\mathcal{R}^2$ satisfying

$$C_i' : |A^T x - b| \leq 1 ; \tag{2}$$
$$A = \sigma_{p_i}^{-1} \begin{pmatrix} -\arctan \phi_i \\ 1 \end{pmatrix} ; \; b = \sigma_{p_i}^{-1} \frac{p_i}{\sin \phi_i} ,$$

where p_i is the distance from the origin to the *LS*-line, σ_{p_i} the uncertainty of the *LS*-line along its normal and ϕ_i the angle of the normal with the x-axis.

The constraint uncertainty γ_i (projection of C_i on the orientation axis) is defined by the interval

$$\gamma_i : \{\theta_i - \sigma_{\theta_i} , \theta_i + \sigma_{\theta_i}\}. \tag{3}$$

where θ_i is the difference of angle between the normal of the map segment and the normal of the *LS*-line and σ_{θ_i} the uncertainty on this difference.
The constraint C_i' is an unbounded set extending till infinity in one direction. The interval γ_i is defined over $\Re$ and is always finite for range type sensors.

4.2 Fusion of uncertainty sets

A classic tool for the fusion of noisy data under a probabilistic approach is the Kalman filter formalism. The principle is to estimate a new consensus $\hat{X}^{k+1}$ by adding to a previous consensus $\tilde{X}^k$ a fraction of a modification ΔX^{k+1} measured by the sensors at time $k+1$ [1][3][5][6]. In a classic implementation, the weighting factor is proportional to the size of the uncertainty on $\tilde{X}^k$ and inversely proportional to the size of the uncertainty on ΔX^{k+1}, respectively described by their covariance matrices. The uncertainty on the consensus is updated as well. Instead, the set membership approach postulates that the uncertainty set resulting from the fusion is best approximated by the intersection in the C-space of all the uncertainty sets of the same feature. The center of the intersection defines a new consensus on the feature. Thus, the problem of fusing the sets comes down to finding the tightest one bounding the intersection.
In our implementation of the set membership principle, the uncertainty sets in the C-space are modeled by their projection on the ground plane and the orientation axis (see Figure 2). The drawback of this approximation is the pawning of the correlation between the robot location and orientation uncertainties. Relations (2) and (3) describe this approximation for a constraining uncertainty set infinite in one degree of freedom. Similarly, the uncertainty set of the robot configuration (an ellipsoid in the C-space) is approximated by the set of points in $\Re^2$ satisfying

$$\tilde{\omega} : (x - \tilde{x})^T \tilde{E} (x - \tilde{x}) \leq \kappa ; \; \kappa > 0 \tag{4}$$

where $\tilde{x}$ is the projection of ellipsoid center on the ground plane and $\tilde{E}$ is the covariance matrix. The uncertainty interval of the projection on the orientation axis is described by

$$\tilde{\epsilon} : \{\tilde{\theta} - \sigma_{\tilde{\theta}} , \tilde{\theta} + \sigma_{\tilde{\theta}}\}. \tag{5}$$

where $\tilde{\theta}$ is the projection of $\tilde{X}$ on the orientation axis.

This approximation greatly simplifies the complexity of the fusion problem, which is split now into a 2D fusion of C_i''s with $\tilde{\omega}$ and a 1D fusion of the γ_i's with $\tilde{\epsilon}$.
Solving the 2D fusion problem comes to finding the weighting factors α_i in

$$\hat{\omega} : \tilde{\omega} + \sum_{i=1}^{M} \alpha_i C_i' \leq \kappa + \sum_{i=1}^{M} \alpha_i \tag{6}$$

that minimize $S(\hat{\omega})$, where $S()$ denotes the surface and M the number of *LS*-lines. The exact ellipse minimizing $S()$ can be solved by numerical methods [1]. However, $\hat{\omega}$ is embedded in $\Re^2$ and the ellipse

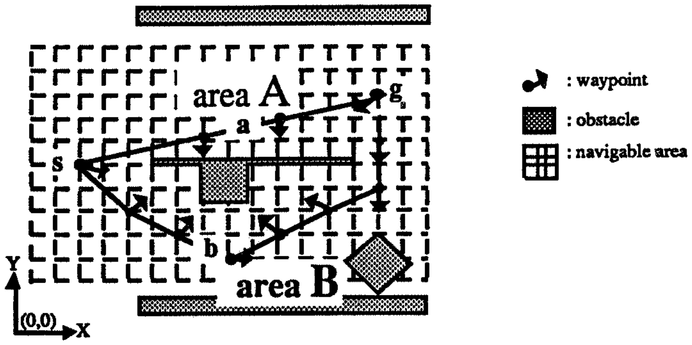

Figure 3: map of the workspace showing two C-paths going from configuration s to g, where a is the shortest one and b is the one provided by the SUF motion path planner.

is the tightest one bounding the intersection. Note that $\hat{\omega}$ alone describes both the uncertainty and the position of $\hat{x}$. The details of the computation of the bounding uncertainty ellipse are reported in Appendix for a simple example.
The 1D fusion problem is simply solved by combining the intervals described in (3) and (5). Let us assume that there is only one strip constraint ($M = 1$). The fusion of intervals γ_i and $\tilde{\epsilon}$ is defined only if $\gamma_i \cap \tilde{\epsilon}$ is not empty, in which case the result of the fusion is $\tilde{\epsilon}$ itself. The new consensus $\hat{\theta}$ of the robot orientation is the center of the interval

$$\hat{\epsilon} : \{ \max(\theta_i^s - \sigma_{\theta_i} , \tilde{\theta} - \sigma_{\tilde{\theta}}) , \min(\theta_i^s + \sigma_{\theta_i} , \tilde{\theta} + \sigma_{\tilde{\theta}}) \}. \tag{7}$$

5 Implementation and experimentations

In this section we present and discuss experimental results obtained using our motion control system. The path tracker has been implemented in a program written in C which runs on a SUN-3/80 workstation. The SUF motion path planner is running on a DEC 5000 workstation. The average processing time needed by the C-estimator for a localization cycle is a few tenths of seconds, while the SUF motion path planner needs a few hours to compute the fields and to extract a C-path.
The workspace consists of a 180 by 120 inches rectangular navigable surface (see Figure 3). The only device used for the localization is a laser range sensor mounted on a Nomad-200 robot. The workspace has been set up so that two distinct areas appears : area A provides only two obstacles to the sensor devices (with edges parallel to the horizontal axis of the workspace); area B contains various differently orientated obstacles. Hence, we expect the C-estimator to be efficient in area B only, within which any observation provides good complementary constraints on the robot configuration (and therefore a better localization ability).
We set up 4 different tests based on the two predefined C-paths a (7 waypoints) and b (9 waypoints): follow b using the C-estimator (test No. 1); follow b using only dead-reckoning (test No. 2); follow a using the C-estimator (test No. 3); follow a using only dead-reckoning (test No. 4). During each test, the robot moves back and forth a or b, until the drift is big enough to eventually make the robot hit an obstacle of the workspace. The values reported in Table 1 are the measured drifts (in inches) separating the actual robot configuration to the waypoint.
Test 3 leads to expected results : the drift along the x parameter of the robot configuration grows in time, since no obstacle along a is able to provide a complementary constraint (non-zero projection on the y-axis) that would help reduce the uncertainty on the robot configuration. However, the drift growing rate is slower than for Test 4 (same route using only odometers) : the observation of the parallel edge (see Figure 3) provides partial information limiting the drift along the y-axis. As expected, the

path b	*Test No.1*								*Test No.2*							
1	0	4		6		5		5	0	5		9		16	x	x
2	2	3	4	5	5	4	6	5	0	4	5	8	9	14	x	x
3	1	3	4	4	5	4	4	5	0	2	4	7	8	10	x	x
4	1	2	3	4	4	4	5	4	1	4	3	7	7	9	†	x
5	2	0	3	2	4	3	3	4	1	4	5	6	6	6	9	x
6	2	0	3	1	2	2	3	3	3	5	5	6	7	6	7	x
7	1	0	2	2	2	3	5	4	4	5	6	6	7	7	7	x
8	1	1	2	3	4	4	4	4	5	4	5	5	8	6	8	x
9	1		2		4		4		5		4		8		9	x
path a	*Test No.3*								*Test No.4*							
1	0	1		4		x	x	x	0	5		x	x	x	x	x
2	0	1	2	3	3	†	x	x	1	5	6	†	x	x	x	x
3	0	1	1	2	3	6	x	x	2	6	6	8	x	x	x	x
4	1	1	1	2	4	5	x	x	2	5	7	8	x	x	x	x
5	1	1	1	3	4	4	x	x	3	6	8	9	x	x	x	x
6	1	2	2	3	3	3	x	x	4	6	8	9	x	x	x	x
7	2		3		3		x	x	6		9		x	x	x	x

Table 1: drift (in inches) of the robot at each waypoint for tests 1, 2, 3 and 4. The dagger sign (†) is reported when the robot collided with an obstacle while a test was performed. It is followed by cross signs (x), indicating that the robot didn't reach the corresponding waypoint.

only combination actually able to move the robot back and forth s and g within a bounded drift is the C-estimator combined with the C-path b (test 1).

6 Conclusion

In this paper we described the parts composing a motion control system dealing with uncertainty in the whole process, from the motion path planning to the path tracking. The experimental runs shows that a proper collaboration within the motion control system is an important aspect in practical navigation. Although the development of this concept is not completed, we believe that this approach provides a way to enhance the reliability of the motion control of a mobile robot inside a complex environment, such as an office. Future developments aim at integrating various sensor devices using the same architecture and splitting the motion control system into parallel processes.

7 Appendix

This appendix details the computation of the tightest ellipse bounding the intersection of a single constraint strip ($M = 1$ in relation (6)) and an ellipse bounding the robot location uncertainty (see relations (2) and (4))
The tightest ellipse satisfies :

$$\hat{\omega} : (x - \tilde{x})^T \tilde{E}(x - \tilde{x}) + \alpha |A^T x - b| \leq 1 + \alpha \; ; \; \alpha \geq 0 \tag{8}$$

This can also be expressed by

$$\hat{\omega} : (x - \hat{x})^T \hat{E}(x - \hat{x}) \leq 1 , \tag{9}$$

where $\hat{x}$ is the center of $\hat{\omega}$ (the new estimate of the robot location).
The ellipse with smallest surface that includes $\tilde{\omega} \cap \mathcal{C}_1$ is the one that minimizes $(\det \hat{E})^{-1}$. According to Deller [9], the corresponding value for α is the most positive value of the second degree polynom

$$G^2\alpha^2 + (3 + b^2 - G)G\alpha + 2(1 - b^2) - G = 0 \, ; \; G = A^T \tilde{E}'^{-1} A \tag{10}$$

The tightest uncertainty ellipse is defined by

$$\hat{E} = KB^{-1} \text{ and } \hat{x} = \alpha b BA , \tag{11}$$

$$\text{where } K = \frac{1}{1 + \alpha(1 - b^2)} + \hat{x}^T B^{-1} \hat{x} \text{ and } B = \tilde{E}^{-1} - \frac{\alpha}{1 + \alpha G} \tilde{E}'^{-1} A A^T \tilde{E}'^{-1} \tag{12}$$

Inconsistencies are easily detected : K is negative if $\tilde{\omega}$ and c^s do not intersect. Furthermore, if $\mathcal{C}_1$ does not reduce the uncertainty in the system ($\tilde{\omega} \cap \mathcal{C}_1$ is $\tilde{\omega}$ itself) then there is no positive solution for α in (10).

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